

Ex: $x + y + z = 1$ $x + 3z = -2 + \lambda$ $x - y + rz = 3$

$$[A|b] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 0 & 3 & -2+\lambda \\ 1 & -1 & r & 3 \end{array} \right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & -3+\lambda \\ 0 & -2 & r-1 & 2 \end{array} \right]$$

$$= \left[\begin{array}{ccc|c} x & y & z & \\ 1 & & & \\ 0 & -1 & 2 & -3+\lambda \\ 0 & 0 & r-5 & \lambda-2\lambda \end{array} \right]$$

1) No sol. if $r-5=0$ but $\lambda-2\lambda \neq 0$
 $r=5$ but $\lambda \neq 4$

2) Infinitely many sol. $r-5=0$ & $\lambda-2\lambda=0$
 $r=5$ & $\lambda=4$

3) Unique sol. $r \neq 5$ $z = (\lambda-2\lambda)/(r-5)$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{REF} \left[\begin{array}{ccc|c} x & y & z & \\ 1 & 0 & 3 & 2 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$z \rightarrow$ free variable

$$z = k$$

$$y - 2z = -1$$

$$\Rightarrow y = -1 + 2k$$

$$x + 3z = 2$$

$$x = 2 - 3k$$

Inverse of Matrix: A square matrix B is said to be inverse of A if $AB = I = BA$

Eg.

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 2 & 4 \\ 1 & 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$R_3 \rightarrow R_3 + \frac{1}{2}R_2$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 6 \\ 0 & 1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow -\frac{1}{2}R_2$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & \frac{1}{2} & 1 \end{bmatrix} A$$

$$R_1 \rightarrow R_1 - 2R_2$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1 & -\frac{1}{2} & 0 \\ -2 & \frac{1}{2} & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 1 & 0 \\ & & & \end{bmatrix}$$